National Center and State Collaborative

NCSC Curriculum Resource to Prepare Students for AA-AAS

Mathematics Content: Ratio and Proportions

Julie Thompson
Diane Browder
Amy Lehew

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Mathematics Content: Ratio and Proportions

The purposes of the Curriculum Resource Guides are:
• To provide guidance for teaching the Florida Standards to students with Significant Cognitive Disabilities that both aligns with these standards and provides differentiation for individual student needs • To provide examples for differentiating instruction for a wide range of SWSCD. These examples can be used in planning specific lessons, alternate assessment items, and professional development.

1a.What is "ratio and proportions" and how is it taught in general education settings?

1a.1 Essential knowledge in this content area

Precursors to Ratio and Proportion

Ratio and proportion are not introduced until the middle grades in the Florida Standards; however, in grades 3-5, students experience and gain proficiency with multiplication and division, which is later, used to solve ratio and rate problems. Examples of these experiences:

- Equipartitioning: "producing equal sized groups (from collections) or pieces (from continuous wholes) as 'fair shares' for each of a set of individuals"[1]. Here are two examples: (1) There are 15 pencils and 5 students in the class. How many pencils does each person get? Or (2) One brownie is shared among 4 people, which results in ¼ brownie per person.
- Multiplicative Comparisons: the maple tree is 3 times as tall as the lilac.
- Unit conversions: 3 ft. = 1 yd.; 6 ft. = 2 yd.; etc.

[1] Confrey, J., Maloney, A. P., Nguyen, K. H., Wilson, P. H., & Mojica, G. F. (2008). *Synthesizing research on rational number reasoning.* Paper presented at the National Council of Teachers of Mathematics Research Presession, Salt Lake City, UT.

Ratios

A ratio is the comparison between two values. The comparison can be part-to-whole (ratio of apples to total amount of fruit in a bowl) or part-to-part (ratio of apples to bananas). Ratios can be expressed in the following forms: $\frac{3}{5}$, 3 to 5, or 3:5. It is important for students to understand that ratios have little meaning unless they know what the ratio represents. They should be able to state: "For every _____ there are _____." (Can also be stated as "___ to ___"; "_____ out of every ____"; "_____ parts to ____ parts").

Students should be provided practice using physical models to make sense of ratios. For example, below is a demonstration of four tires for every one car (4:1). Students are asked, "How many tires if you have three cars?"

Explain to students that by answering the question above they are creating an equivalent ratio. This comparison doesn't change if the values are *varied multiplicatively by the same factor*. A ratio of 6:4 holds true when multiplying both quantities by a given number. For example, 6:4 = 12:8 (multiplied both by 2) or 18:12 (multiplied both by 3).

The ratio also holds true if both numbers *are divided by the same number* or *split* by the same number. 6:4 = 3:2 (both numbers divided by 2).

Equivalent ratios can be created by multiplying both numerals in a ratio pair by the same positive number. Following with the ratio 4:1, to determine the equivalent ratio that results in 3 cars, students would multiply each number in the ratio pair by 3. So, 4:1 = 12:3. Thus, for every 12 tires there are 3 cars, or vice versa, for every 3 cars, there are 12 tires.

Below is an example of a more complex problem:

A store has a sale. T-shirts are 2 for $18. What would 6 shirts cost? How many shirts could you buy if you have $45? [Solution: To find the price of one T-shirt divide $18 by 2. One shirt costs $9. Multiply $9 times 6 to get $54. To find the number of shirts that can be purchased with $45, divide by $9. Five shirts can be purchased with $45.]

Break the problem into two parts. Start with the first question, "What would 6 shirts cost?" The student has been given the ratio 2:18. In the same way that equivalent ratios can be determined by multiplying both numerals in a ratio pair by the same number, they can also be determined by dividing both numerals in a ratio pair by the same number. To

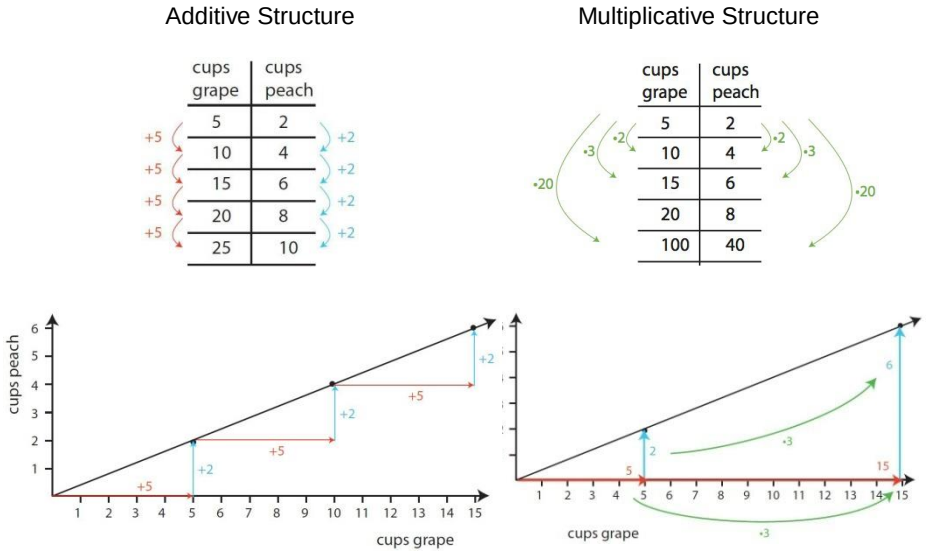
solve the problem of determining the cost of 6 shirts, it would be helpful to find the cost of 1 shirt by dividing both numerals by 2.

2:18 = [2÷2]:[18÷2]= 1:9
So, 1 shirt costs $9

We want to know the cost of 6 shirts so we will multiply the numbers in the ratio pair by 6.
1:9 = [1x6]:[9x6]= 6:54
The answer to the first question is: 6 shirts cost $54.

<u>Ratios can be displayed using a table or graph:</u>

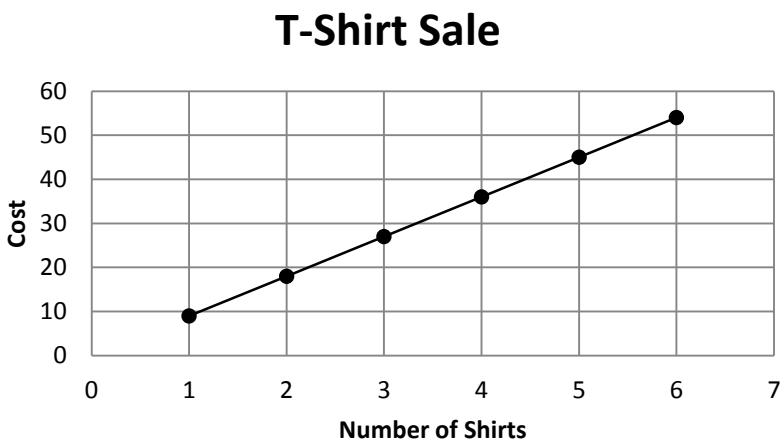
Showing structure in tables and graphs[2]



In the tables, equivalent ratios are generated by repeated addition (left) and by scalar multiplication (right). Students might be asked to identify and explain correspondences between each table and the graph beneath it (MP1).

<u>Ratios can be plotted as ordered pairs:</u>
Using the example above, we can find the points to plot by using the ratio [number of shirts: cost]. The number of shirts will be the x-coordinate and the cost will be the y-coordinate in the ordered pair. We found that 1 shirt costs $9, or 1:9, so our first ordered pair is (1,9). We can find the remaining ordered pairs by multiplying both numerals in the ratio pair by the same number.

[2] Retrieved from http://commoncoretools.files.wordpress.com/2012/02/ccss_progression_rp_67_2011_11_12_corrected.pdf, p.6.



Ratios can be used in problem solving by thinking about the total amount for each ratio unit.

For example:

The ratio of smores to marshmallows is 1:2 and the camp counselor is making 21 smores, how many marshmallows does she need? [Solution: Student recognizes there are twice as many marshmallows as there are smores and multiples the total number of smores by 2. The answer is that she will need 42 marshmallows.]

Proportions

A proportion is an equation stating that two ratios are equivalent. Students should be given opportunities to determine if two ratios are "proportional" (e.g., 1:2 and 3:6) or "not proportional" (e.g., 2:3 and 4:5).

The graph below demonstrates proportional relationships:

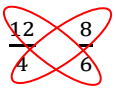
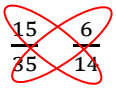
cups grape	cups peach
5	2
10	4
15	6
20	8
25	10

+5 +5 +5 +5 +2 +2 +2 +2

Using these relationships proportions can be written such as 5:2 = 15:6 or 10:4 = 20:8.

In practice students might be given ratios and asked to determine if they are equal (proportional) or if one ratio is greater than or less than the other (not proportional).

There are several ways students can determine if two ratios are proportional. Below are three examples of how to do so:

Ratios	**12:4 and 8:6**	**15:35 and 6:14**
Writing as fractions and determining common denominator	$\frac{12}{4}$ and $\frac{8}{6}$ $\frac{36}{12} \neq \frac{16}{12}$	$\frac{15}{35}$ and $\frac{6}{14}$ $\frac{3}{7} = \frac{3}{7}$
Cross-multiplying	$\frac{12}{4}$ and $\frac{8}{6}$  12 x 6 and 4 x 8 72 ≠ 32	$\frac{15}{35}$ and $\frac{6}{14}$  15x14 and 6x35 210 = 210
Dividing to see if the quotients are equal	12÷4 and 8÷6 3 ≠ 1.33	15÷35 and 6÷14 .43 = .43
Proportion	12:4 ≠ 8:6 (not proportional)	15:35 = 6:14

1a.2 Common misunderstandings in this content area

- Students may not understand that, for example 3:9 is the same ratio as 1:3.
- Students may wrongly use direct division instead of proportional division in calculations. For example, if it takes 4 people 2 hours to decorate 200 cupcakes they might answer that it takes 2 people 1 hour. [The answer is that it would take 2 people 4 hours.]
- Students may use the equal sign incorrectly when describing proportional relationships. For example for the ratio 2:5 or "for every 2 there are 5" a student might incorrectly represent this by writing 2=5.

1a.3 Prior knowledge/skills needed (can be taught concurrently)

- Multiplication
- Division
- Using calculator to multiply and divide
- Identify and use >, <, and = symbols to compare values

2. What are some of the types of activities general educators will use to teach this skill?

2.1 Activities from General Education Resources

- Use manipulatives to act out problems such as “There are 24 shells and three children. How many shells are there for each child?” or “Alyssa, Heidi, and Grant found 24¢. The want to share it equally. How many cents should each child get?”[3]
- Give students several problems where they must determine which ratio is greater. Allow them to use a calculator to find the quotients to compare if needed.[5]
- Give students several ratios and ask them to write each ratio in its simplest form.[4]
- Ask students to draw a picture showing 4 pencils and a number of pens in which the ratio of pencils to pens is 2:3.
- Have students use a proportion to solve the following problem: “Out of the 32 students in health class, 24 prefer using gel toothpaste. Based on these results, how many of the 500 students in the school can be expected to prefer using gel toothpaste?”[4]

Links Across Content Areas

- Literature:
  - Read *The Doorbell Rang* by Pat Hutchins and have students illustrate how an increasing number of children can share cookies.[3]
  - Read the poem *Jimmy Jet and his TV set* by Shel Silverstein and ask students to create ratios to compare time spent watching TV to time spent doing other activities.[5]
- Social Studies:
  - Ask students how many full terms Franklin D. Roosevelt served if he was president for 12 years. (Remind them that one term equals 4 years.)[3]
  - Have students locate the scale on the map to create a proportion in order to calculate the distance between two cities. For example, if Charlotte is 8 inches from Raleigh and the scale is 2 inches equals 45 miles, what is the distance from Charlotte to Raleigh?[4]
  - Explain to students that the ratio of the heads carved on Mount Rushmore is 1:12 or 1 inch to 12 feet. Ask students how they would use this knowledge to determine the actual size of the president’s heads.

[3] Willoughby, S. S., & SRA/McGraw-Hill. (2003). *SRA math: Explorations and applications*. Worthington, Ohio: SRA/McGraw-Hill.
[4] Bailey, R., Day, R., Frey, P., Howard, A. C., Hutchens, D. T., McClain, K., Moore-Harris, B. Glencoe/McGraw-Hill. (2004). *Mathematics: Applications and concepts*. New York: Glencoe/McGraw-Hill.
[5] Willoughby, S. S., & SRA/McGraw-Hill. (2003). *SRA math: Explorations and applications*. Worthington, Ohio: SRA/McGraw-Hill.

3. What Florida Standards Access Points Are Addressed in Teaching "Ratio and Proportions"?

Grade Differentiation	Access Points
3rd Grade	MAFS.3.OA.4.AP.8a Solve and check one-step word problems using the four operations within 100.
	MAFS.3.MD.2.AP.3b Select the appropriate statement that compares the data representations based on a given graph (picture, bar, line plots).
4th Grade	MAFS.4.OA.1.AP.2b Determine the number of sets of whole numbers, ten or fewer, which equal a dividend.
	MAFS.4.OA.1.AP.1a Use objects to model multiplication involving up to five groups with up to five objects in each, and write equations to represent the models.
5th Grade	MAFS.5.NBT.2.AP.6a Find whole number quotients up to two dividends and two divisors.
	MAFS.5.NBT.2.AP.6b Find whole number quotients of whole numbers with up to two-digit dividends and two-digit divisors.
6th Grade	MAFS.6.RP.1.AP.1b Describe the ratio relationship between two quantities for a given situation using visual representations.
7th Grade	MAFS.7.RP.1.AP.2a Identify the rate of change/proportional relationship of a linear equation that has been plotted as a line on a coordinate plane..
	MAFS.7.RP.1.AP.2b Identify lines plotted on a coordinate plane that represent a proportional relationship.
	MAFS.7.RP.1.AP.3a Solve word problems involving ratios.
	MAFS.7.G.1.AP.1a Draw pairs of proportional polygons on graph paper.
	MAFS.7.RP.1.AP.1a Solve one-step problems involving unit rates associated with ratios of fractions.
	MAFS.7.RP.1.AP.3b Find percentages in real-world contexts.
8th Grade	MAFS.8.F.2.AP.5c Describe or select the relationship between two plotted graphs.
High School	MAFS.912.F-LE.1.AP.1a Select the appropriate graphical representation of a linear model based on real-world events.

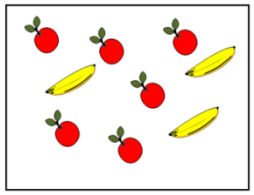
<table>
<tr><th colspan="4">Performance Examples for Priority Access Points</th></tr>
<tr><td colspan="4">Grade 3</td></tr>
<tr><th>Access Points</th><th>Performance Example</th><th colspan="2">Essential Understandings: Concrete Understandings and Representations</th></tr>
<tr><td>Numbers:
MAFS.3.OA.4.AP.8a
Solve and check one-step word problems using the four operations within 100.</td><td>Student selects equation that matches word problem, solves equation, then selects equivalent equation that can be used to check work.

Kunius had 3 weeks to sell cookies for school. He sold 6 boxes each week. Which of these will show how many boxes Kunius sold?

3 + 6 3 x 6 3 – 6
How many boxes did Kunius sell?

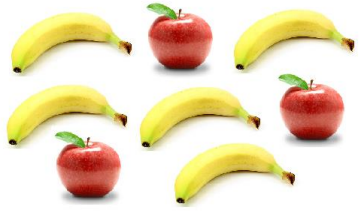
Which one of these can be used to check your work?

3 + 3 + 3 18 – 3 6 + 6 + 6</td><td>Concrete Understandings:

• Combine (+), decompose (-), and multiply (x) with concrete objects; use counting to get the answers.
• Match the action of combining with vocabulary (i.e., in all; altogether) or the action of decomposing with vocabulary (i.e., have left; take away; the difference) in a word problem.</td><td>Representation:
• Draw or use a representation of the word problem.
• Add on or count back depending upon the words in the problem.
• Understand the concepts, symbols, and vocabulary for: +, =, -, x.</td></tr>
<tr><td>Numbers:
MAFS.3.MD.2.AP.3b
Select the appropriate statement that compares the data representations based on a given graph (picture, bar, line plots).</td><td>Student selects symbol to compare two numbers.
A) Are there more apples or bananas in this picture?
 
B) Tell me how you know that there are more apples? (There are 6 apples and only 3 bananas)
C) These symbols are used to compare numbers. This is equal to (point to =), this is greater than (point to >), and this is less than (point to <). Put the symbol in the blank that shows there are more apples.
6 apples ____ 3 bananas = > <</td><td>• Use an object to identify which category has more on a bar graph.</td><td>• Understand the vocabulary of more, less, least, most, same, equal.

• Identify visuals used to represent data in a graph.</td></tr>
</table>

Grade 4			
Access Points	**Performance Example**	**Essential Understandings: Concrete Understandings and Representations**	
Numbers: MAFS.4.OA.1.AP.2b Determine the number of sets of whole numbers, ten or fewer, which equal a dividend.	"Bethany and her friends decided to start a dog walking business after school to earn some spending money. Bethany has three friends, for a total of four people who want to walk dogs." Point to the picture of Bethany and her friends (provide a picture of four girls). "There are eight dogs that need to be walked." Point to the picture of the dogs (provide a picture of eight dogs). "Each person will walk an equal number of dogs. How many dogs will each person walk? You can use these blocks to help solve the problem."	**Concrete Understandings:** • Use manipulatives to separate sets. • Count a set of objects within 10 • Group a set of objects into equal sets (division).	**Representation:** • Understand the following vocabulary: divide, separate, total, etc. • Identify or draw a pictorial representation of the problem.
Patterns: MAFS.4.OA.1.AP.1a Use objects to model multiplication involving up to five groups with up to five objects in each, and write equations to represent the models.	Present a paper with the following printed on it and read it aloud: "Ms. Smith is an art teacher. She is preparing to teach an art lesson to five students. Each student will need four markers to complete the art activity. You need to find out how many markers Ms. Smith will need all together." Give the student 24 markers. "Use these markers to show me how five students would each get four markers. You may not use all the markers." If the student makes an error, model the correct answer and say "There should be five groups of four markers, like this." "How many markers does the teacher need all together?"	**Concrete Understandings:** • Create an array of sets (e.g., three rows of two objects) from a set of objects. • Count a set of objects within 25.	**Representation:** • Understand the following vocabulary and symbols: multiplication (x), division (÷), equal (=). • Use graph paper or draw an array that has up to five columns and up to five rows.

Grade 6			
Access Point	**Performance Example**	**Essential Understandings: Concrete Understandings and Representations**	
Patterns: MAFS.6.RP.1.AP.1b Describe the ratio relationship between two quantities for a given situation using visual representations.	Show students the visual (below) and say: “A ratio is a comparison between two different amounts. Here is a picture of the fruit that the cafeteria is offering to students. There are apples and bananas.” Point to an apple and a banana. Ask the student: “What is the ratio of apples to bananas?” After they respond, then ask: “What is the ratio of bananas to the total amount of fruit?” 	**Concrete Understandings:** • Match/identify a simple ratio (1: X) to the relationship between two quantities. • Given a situation, use objects or calculate to set up a ratio.	**Representation:** • Recognize the meaning of the placement of numbers in a ratio for a given situation. • Show a ratio in three ways: number to number (1 to 2) expressed as a fraction (1/2) or using a colon 1:2. • Represent the ratio of objects (e.g., red hats) to the total number of objects (red and green hats), part-to-whole. • Represent the ratio of the number of one object (red hats) to the number of other objects (green hats) from a set of objects (red and green hats), part-to-part. • Understand the following concepts and vocabulary: ratio, rate, proportion, prices, portions per person.

<table>
<tr><td colspan="4">Grade 7</td></tr>
<tr><td>Access Point</td><td>Performance Example</td><td colspan="2">Essential Understandings: Concrete Understandings and Representations</td></tr>
<tr><td>Numbers:
MAFS.7.RP.1.AP.2a
Identify the rate of change/proportional relationship of a linear equation that has been plotted as a line on a coordinate plane.</td><td>Show the students the table (below) and read them the following “Mr. Gowen’s class is selling pizza to the middle school’s sports teams to raise money for a field trip. Each pizza has 12 slices. They need to buy enough pizzas to make sure everyone gets enough. Look at this table. In this column are the different types of sports, in this column are the number of players on each team, and in this column are the number of pizzas needed for each team. If there are 48 players, we would need 12 pizzas, if there are 24 players, we would need 6 pizzas, if there are 12 players we would needs three pizzas. How many pizzas would be needed to feed the golf team, which has 8 players?”
<table>
<tr><th>Sports Team</th><th>Number of Players</th><th>Number of Pizzas Needed</th></tr>
<tr><td> 
Football</td><td>48</td><td>12</td></tr>
<tr><td> 
Baseball</td><td>24</td><td>6</td></tr>
<tr><td> 
Soccer</td><td>12</td><td>3</td></tr>
<tr><td> 
Basketball</td><td>8</td><td>____</td></tr>
</table>
Present the next question and response options and ask: “What is the relationship between the number of pizzas and the number of players?”
<table>
<tr><td>1 pizza for 48 players</td><td>1 pizza for 4 players</td><td>1 pizza for 10 players</td></tr>
</table></td><td>Concrete Understandings:
• Recognize the constancy of one object to its parts (i.e., one face: two eyes).
• Recognize the meaning of the placement of numbers in a proportion for a given situation.
• Represent the proportion of objects (e.g., red hats) to the total number of objects (red and green hats).
• Identify the proportional relationship using visuals.</td><td>Representation:

Recognize the meaning of the placement of numbers in a proportion for a given situation.

Represent the proportion of objects (female students) to the total number of objects (students in class), part-to-whole.

Represent the proportion of the number of one object (female students) to the number of other objects (male students) from a set of objects (male and female students), part-to-part.

Find a percentage of a quantity as a rate per 100 (e.g., 20% of a quantity means 20/100 or .20 times the quantity).

Understand the following concepts, symbols, and vocabulary: proportion, ratio, rate, prices, portions per person.</td></tr>
</table>

<table>
<tr>
<td>Numbers: 7.NO.2f1: Identify the proportional relationship between two quantities (7.RP.2)</td>
<td>Show student the table (below) and say the following: “A proportional relationship is a relationship between two numbers where one number is a constant multiple of the other number. An example is the number of dimes in a dollar. There are always 10 dimes in 1 dollar. No matter how many dollars there are, there are always 10 dimes in one dollar.” Show student the example below.

Number of dimes

Number of dollars

10 dimes for 1 dollar

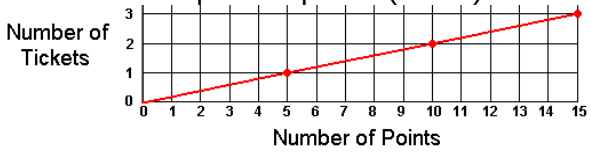
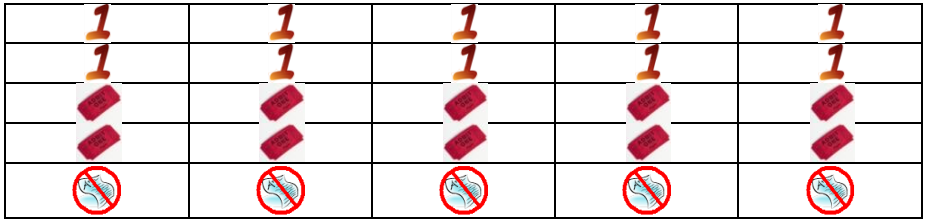
Remove the previous visual and show students the new visual and word problem (below) and say: Jada delivers cases of water to different classrooms in the school. She delivered 4 cases of water (96 bottles of water) to Ms. Smith’s class, 3 cases of water to Mr. Palmer’s class (72 bottles of water), and 2 cases of water to Mr. Dragoo’s class (48 bottles of water). What is the proportional relationship between cases of water and bottles of water?” Give student the response options to pick from.

<table>
<tr><th>Class</th><th>Cases of Water</th><th>Bottles of Water</th></tr>
<tr><td>Ms. Smith</td><td>4</td><td>96</td></tr>
<tr><td>Mr. Palmer</td><td>3</td><td>72</td></tr>
<tr><td>Mr. Dragoo</td><td>2</td><td>48</td></tr>
</table>

What is the proportional relationship between cases of water and bottles of water?

<table>
<tr><td>24 bottles to 1 case</td><td>96 bottles to 1 case</td><td>48 bottles to 1 case</td></tr>
</table>
</td>
<td>Concrete Understandings:
• Recognize the constancy of one object to its parts (i.e., one face: two eyes)</td>
<td>Representation:
• Recognize the meaning of the placement of numbers in a proportion for a given situation
• Show a proportion/ratio in three ways: number to number (2 to 6) expressed as a fraction (2/6) or using a colon 2:6
• Represent the proportion of objects (e.g., female students) to the total number of objects (students in class) Part-to-whole.
• Represent the proportion of the number of one object (female students) to the number of other objects (male students) from a set of objects (male and female students) Part-to-part.
• Understand concept, symbols and vocabulary: proportion, ratio, rate, prices, portions per person</td>
</tr>
</table>

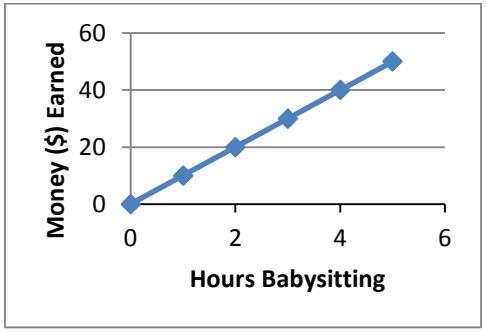
Numbers: MAFS.7.RP.1.AP.3a Solve word problems involving ratios.	Show the student the following word problem and read it aloud.  Jesse mows his neighbors' lawns to earn extra money. He mows 2 lawns an hour. That means the ratio of lawns mowed per hour is 2 to 1. A: Present the following question and read it aloud: "Jesse mowed lawns for 2 hours on Saturday morning. How many lawns did he mow?" B: Present the following question and read it aloud: "Jesse mowed lawns for 3.5 hours Saturday afternoon. How many lawns did he mow in the afternoon?" C: Present the following question and read it aloud: "If Jesse mowed 4 lawns in the morning and 7 lawns in the afternoon, how many lawns did he mow all together?"	**Concrete Understandings:** • Show rate when asked • Show proportion when asked • Select a set for the ratio given	**Representation:** • Locate information within a word problem • Make meaning of a word problem • Understand concept and vocabulary: ratio
Patterns: MAFS.7.RP.1.AP.3b Find percentages in real-world contexts.	Give student the visual (below) and read it aloud to them: • There are 600 students who attend Smith Middle School. • 40% of the students are 7th graders. Use a proportion to solve for how many students are 7th graders. $\frac{x}{600} = \frac{40}{100}$	**Concrete Understandings:** • Identify how one variable changes in relation to another variable in a directly proportional relationship ○ *a/b = c/d* (If *a* increases, what will happen to *c*?)	**Representation:** • Use a proportion method to solve (part/whole = number/100) • Set up a proportion to solve for an unknown value • Follow a sequence of steps to solve a problem • Understands vocabulary and symbol: % (percent), proportion

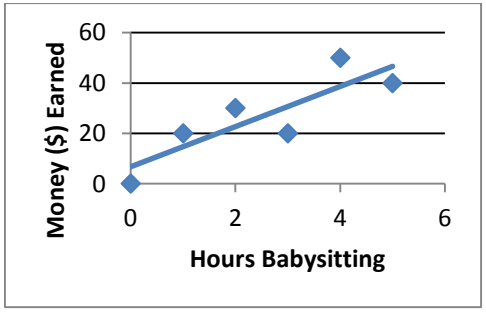
Grade 8			
Access Point	**Performance Example**	**Essential Understandings: Concrete Understandings and Representations**	
Numbers: MAFS.8.F.2.AP.5c Describe or select the relationship between two plotted graphs.	Show the student the following graph and read: "Mr. Tate's class earns points for good behavior. This graph shows how many points need to be earned to trade in for one ticket. This y-axis shows the number of tickets that each student has. This x-axis shows the number of points the student has earned. How many points must a student earn in order to earn one ticket?" Present response options (below).  9 | 3 | 5 After student responds, read the next part: "This table shows the number of tickets needed to get a homework pass. This column shows the number of tickets needed and this column shows the number of homework passes." Read the table to the student. **Tickets** | **Homework Passes** 3 | 1 6 | 2 9 | 3 5 Points = 1 Ticket 3 Tickets = 1 Homework Pass Present Point Icons, Tickets Icons, and Homework Pass Icons (see below: individually cut up), and say "How many points do you need to get one homework pass?" Let students use icons to help them solve the problem. 	**Concrete Understandings:** • Recognize the quantity in two sets, without counting, to determine which is relatively larger.	

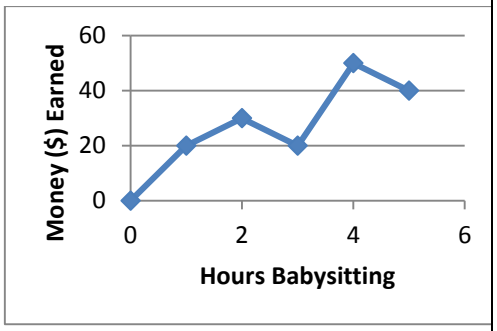
<table>
<tr><td colspan="4">High School</td></tr>
<tr><td>Access Point</td><td>Performance Example</td><td colspan="2">Essential Understandings: Concrete Understandings and Representations</td></tr>
<tr><td>Patterns: MAFS.912.F-LE.1.AP.1a

Select the appropriate graphical representation of a linear model based on real-world events.</td><td>Show the students the following word problem and read it aloud:

Abbey earns money babysitting. For every hour she works, she makes $10. For 2 hours of babysitting, she made $20. For 3 hours of babysitting, she made $30.
y = 10x

Which line graph shows the relationship between the number of hours babysitting and the money she makes?
 




</td><td>Concrete Understandings:
Match a point not on a line as not being part of a data set for a given line</td><td>Representation:
Identify coordinates (points) on a graph and in a data table
Select a graph that represents a simple linear equation
Match or plot the points from a data table on a graph
Understand concepts and vocabulary: x-axis, y-axis, x intercept, y intercept, line, slope</td></tr>
</table>

4. What are Some Additional Activities That Can Promote Use of this Academic Concept in Real World Contexts?

- Using maps, blue prints, and/or scale models require understanding proportions. When a student reads a map, they need to understand that the map is a proportion of the actual area represented. Have student use the key to determine the proportion of the map to the area represented.
- Have students apply proportions by calculating the dosage of a medicine for a given weight of a person (or of themselves).
- Ask students to increase a recipe to make four times as much and apply proportions to the measurements in the recipe.
- Ask student to calculate the total number of forks and napkins needed to set a place for four people at a total of 24 tables.
- Take students to the grocery store and select two similar products that have different prices and ask them to determine which item costs less per ounce.[4]
- Have students go through the newspaper to find a sales ad for a grocery store and locate an item that is 2 for 1. Have them locate the item to find the price of one item and then calculate how much they would pay to purchase 8 items.[4]
- Teach students to use a ratio to calculate tips and have them practice calculating tips for different situations (hair cut, taxi ride, restaurant, etc.).

5. How Can I Further Promote College and Career Readiness when Teaching "Ratio and Proportion"?

Ideas for Promoting Career/ College Ready Outcomes

Communicative competence

Students will increase their vocabulary to include concepts related to "ratio and proportion," and should be able to read a ratio using correct terminology. In addition, students will be learning concepts such as: "equal", "every", "part", "same", and "different."

Fluency in reading, writing, and math

Students will have opportunities to increase their numeracy and sight word fluency while participating in problem solving related to "ratio and proportion" such as number recognition, counting, one-to-one correspondence, and reading concepts that include the use of an understanding of equal and same.

Age appropriate social skills

Students will engage in peer groups to solve problems related to "ratio and proportion" that will provide practice on increasing reciprocal communication and age appropriate social interactions. For example, students might work together with their peers to find the distance between cities on a map using scale drawings.

Independent work behaviors

Ratios and proportions are used in many real-life work settings. For example, janitors may need to mix chemicals (ratio of bleach to water), hair stylists need to mix hair chemicals for permanents and coloring, lawn maintenance workers may mix weed killer,

bricklayers need to understand ratios to make up mortar, child care workers and nurses assistants may need to understand ratios for mixing things like juice and water, gardeners and construction workers need to understand scale drawings, and bakers and chefs may need to adjust recipes to account for increases in serving size. By solving real life problems related to “ratio and proportion” students will improve work behaviors that could lead to employment.

Skills in accessing support systems

At times, students will need to ask for assistance to complete activities related to “ratio and proportion” which will give them practice in accessing supports. Students will gain practice asking for tools such as calculators. They can ask a peer to complete the physical movements of the tasks they are not able to do themselves. Be sure to teach students to ask versus having items or supports automatically given to them.

6. How Do I Make Instruction on “Ratio and Proportion” Accessible to ALL the Students I Teach?

6.1 Teach Prerequisites and Basic Numeracy Skills Concurrently: Remember that students can continue to learn basic numeracy skills in the context of this grade level content.

Basic numeracy skills that can be worked on as a part of a lesson relating to ratio and proportions:

- Number identification
- Counting
- One-to-one correspondence
- More/less
- Greater than/less than
- Same and equals

6.2 Incorporate UDL: Universal Design of Learning When Teaching Area

Some examples of options for teaching Measurement and Geometry to students who may present instructional challenges due to:				
	Sensory Differences such as Blindness, Visual Impairment, Deafness, or Deaf/Blindness	**Physical Disability or Motor Differences (such as weakness or motor planning difficulty)**	**Extremely limited evidence of experience/ skill or motivation/ attention.**	**Lack of or extremely limited use of speech.**
Options for Representation	**Provide auditory options** – Talking calculator – Text-to-speech software or voice recordings to read aloud story problems – Single message sequence voice–output devices to count aloud – Captioning software that presents auditory information visually **Provide tactile options:** – Object cues, using - miniature objects or other tangible symbols to assist with problem comprehension and operations – Create numbers and symbols out of tactile materials – When demonstrating graphical linear representations of ratios, raise the grid by using glue over grid, puffy paint, or wiki sticks to represent proportional relationship (Students will need the grid raised to count x- and y-axis and another texture or height for the line representing the proportional relationship). **Provide visual and manipulative options to scaffold representation of concepts:** – Color code problems and corresponding parts of calculator to support students correctly entering information – Have students physically demonstrate ratios using manipulatives or concrete objects (e.g., pencils to students).	**Reduce Physical Effort** – Place materials on slant board or eye gaze array – Display flip chart, interactive white board or other teaching materials at student eye level – Student can scan an array of possible options and use a switch to select the answer – Use computer representation of figures that can be manipulated with switch – Demonstrating ratios verbally (e.g., 4: 3 can be demonstrated by saying beep beep beep beep: bam bam bam)	**Illustrate through multiple media** – Utilize interactive whiteboard – Incorporate interactive websites that provide nonlinguistic tools for exploring math concepts: Illuminations http://illuminations.nctm.org/ActivitySearch.aspx Math Open Reference http://www.mathopenref.com/ There are many resources listed here: http://www.udlcenter.org/implementation/examples – Use a talking calculator	**Provide customized display of information** – Consistent model by utilizing modes of communication used by students (point to symbols representing concepts, operations) – Teacher model competent use of AAC during instruction

	Sensory Differences such as Blindness, Visual Impairment, Deafness, or Deaf/Blindness	Physical Disability or Motor Differences (such as weakness or motor planning difficulty)	Extremely limited evidence of experience/ skill or motivation/ attention.	Lack of or extremely limited use of speech.
Options for Expression	**Vary the methods for response by:** – Student states answer or scans raised numbers to select correct answer; use voice output devices for student to select the correct answer – Provide manipulatives for student to respond or contribute to interaction – Student states answer by selecting picture or symbol. – Students can use talking calculator to assist with counting number of items in each ratio unit. – Give students a ratio with two different types of counters/ manipulatives for each ratio unit (i.e., 2:4 would be represented with 2 pennies and 4 bear counters). Have students create the same ratio with two different types of counters (i.e., 2 paper clips and 4 pencils). Provide student with response options or AAC device.	**Provide options for responses/expression:** – Student selects numbers versus writing them – Selection of correct answer is done after a model – Ratios can be recognized without counting. Display ratios that are the same and one not the same and use the example/non example script to help students identify similar ratios. – Rather than indicating ratios by number, with simple ratios, have them indicate proportional relations by stating same or not same. **Optimize access to tools/ alternatives for responding:** – Provide symbols, objects, manipulatives, and pictures for matching/ student responses – Use computer/interactive whiteboard to show ratios so student can interact using a switch or eye gaze.	**Provide multimedia options for responses/expression:** – Allow the student to make selections by pointing to, gazing at, or selecting answers on the interactive white board – Utilize a switch or adapted computer mouse – Have student write answers with novel pencil or use a tablet computer – Students can demonstrate understanding of ratios by eye gazing to proportional ratios, using technology to create ratios (computer games or lessons that can be completed using a switch)	**Provide options for modes of communication:** – Incorporate responses into student's AAC device or eye gaze array – Phrase questions so that they require a "yes/no" response, these can easily be answered using an eye gaze, head turn, two switches, etc. – Choose response by pointing to or selecting object or item – Use a blink response select answer

	Sensory Differences such as Blindness, Visual Impairment, Deafness, or Deaf/Blindness	Physical Disability or Motor Differences (such as weakness or motor planning difficulty)	Extremely limited evidence of experience/ skill or motivation/ attention.	Lack of or extremely limited use of speech.
Options for Engagement	**Recruit interest by providing choices:** – Digital /talking representations, videos, talking calculators – Interactive websites **Increase personal relevance:** – Use items that are familiar and reinforcing to students. – Incorporate high preference items into story problems, as well as student names **Provide tactile options for engagement:** – Use concrete items of interest to demonstrate ratios and try to incorporate tactile surfaces for students with visual impairment	**Recruit interest by increasing personal relevance:** – Ensure that engaging and high preference content is visible and accessible to student – Use figures that are large enough to accommodate the movements that the student is able to make – Pair student with another student without a physical impairment and have them complete hands on activities together – Use items of high interest when demonstrating ratios. – Have student do a scavenger hunt with favorite items to locate constant ratios (e.g., if student enjoys Legos, have her collect several Lego bricks that have the same number of studs).  1 brick : 8 studs	**Recruit interest by providing choices:** – Digital /talking representations, videos, talking calculators **Provide options for sustaining effort and persistence:** – Break tasks down to maximize student attention – Use high interest items that demonstrate a constant ratio such as a favorite car (1 car 4 wheels), robot (1 robot two extending arms), or DVD (1 DVD to one hole in the center of the disk). – Vary demands and materials to maintain interest **Increase personal relevance:** – Use items that are familiar and reinforcing to students. – Incorporate high preference items into story problems, as well as student names – Use personally relevant items. Consider a tangible token economy system which follows a ratio (e.g., for every 3 tokens, Johnny earns five minutes of listening to Bob Dylan; use CD's to represent minutes).	**Recruit interest with modes of communication:** – Allow students to choose items or subjects that are relevant to them via AAC devices, symbols, or eye gaze array

Promoting Career and College Readiness	Standards for Mathematical Practice
1 Communicative Competence	1 Make sense of problems and persevere in solving them.
2 Fluency in reading, writing, and math	2 Reason abstractly and quantitatively.
3 Age appropriate social skills	3 Construct viable arguments and critique the reasoning of others.
4 Independent work behaviors	4 Model with mathematics
5 Skills in accessing support systems	5 Use appropriate tools strategically.
	6 Attend to precision.
	7 Look for and make use of structure.
	8 Look for and express regularity in repeated reasoning